

# WESTMINSTER SCHOOL THE CHALLENGE 2018 

## MATHEMATICS II

## Tuesday $1^{\text {st }}$ May 2018

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.
All your working should be clearly shown.
You should attempt all the questions.
Please write in black or blue ink.

1 Alex had $£ 51.35$ when he set out to the shops. He paid for seven muffins at $£ 1 \cdot 19$ each and eleven giant cookies. When he got home, he had $£ 27.84$ left. How much was each cookie?

2 Sam ran 800 metres in 2 minutes and 28 seconds. Tom ran 1500 metres in 4 minutes and 37 seconds. Which of them ran faster on average? Show the working you do to decide.

3 a i What is the result of adding $\frac{1}{2}(x+1)$ and $x+\frac{1}{2}$ ?
ii By what would you need to multiply $\frac{10 b}{a}$ to make $\frac{5 a}{b}$ ?
b Make $V$ the subject of

$$
P=\frac{R T}{V-b}
$$

c Solve the equation

$$
\frac{x+10}{3}-2(x-1)=3 .
$$

4 The diagram shows a cylinder with a height of 21 cm and a volume of $1336 \mathrm{~cm}^{3}$. What is the surface area of the cylinder?


5 a The volume of a meteorological balloon increases by $14 \%$ for each 1000 metres it rises through the atmosphere. The volume of the balloon is $5488 \mathrm{~cm}^{3}$ at a height of 3000 metres. What was its volume at ground level?
b The pressure in a meteorological balloon decreases by the same percentage, $P$, for each 1000 metres it rises through the atmosphere. The pressure in the balloon is 101300 Pascals at ground level and 34800 Pascals at a height of 8000 metres.
What is the pressure in the balloon at a height of 5000 metres?
[The Pascal is a unit of pressure equal to one Newton per square metre].

6 There are 2018 students in a school. Every student must study either History or Geography, but students can study both subjects. The head teacher knows that between $80 \%$ and $85 \%$ of the students study History, and that between $30 \%$ and $40 \%$ study Geography.
a What are the maximum and minimum numbers of students who study Geography?
b What are the maximum and minimum numbers of students who study both subjects?

7 The tuck shop sells sandwiches, which all cost the same amount.
David buys eight sandwiches, and pays between $£ 13$ and $£ 14$ for them.
a If a sandwich costs $K$ pence, what is the range of possible values of $K$ ?
Ezra buys eleven sandwiches, and pays between $£ 17$ and $£ 18$ for them.
b Find the cost of a sandwich.

8 The diagram shows how nine squares have been fitted together to form a rectangle.
The smallest square (black) has side length 2 cm and the second smallest square (shaded) has side length $x \mathrm{~cm}$.
a Find the side lengths of the other squares, labelled A to G , in terms of $x$.
b Find the value of $x$, and hence the length and width of the rectangle.


9 A Subonacci list of numbers is formed as follows:

- the first two numbers in the list are given
- each subsequent number is one less than the sum of the previous two.

For example, a Subonacci list could start as follows
26712
because $7=2+6-1$ and $12=6+7-1$.
The first number in a Subonacci list is 4 and the second is 7 .
a Find the next four numbers in the list.
b Is the $2018^{\text {th }}$ number in the list even or odd? Justify your answer.
c Prove that none of the numbers in the list is a multiple of three.

10 The diagram shows two straight roads, one heading due North and one heading due East from point O .


Car A heads North and Car B East; they leave O simultaneously.
Car A travels at 27.4 metres per second, and Car B travels at 30.8 metres per second.
a How long does it take before Car A and Car B are 1000 metres apart?
When they are 1000 metres apart, Car A and Car B turn and head directly towards each other, at the same speeds as before.
b How long is it before they collide?

11 a The diagram shows triangle PQR. Angle PQR is a right angle

$P R=5.3 \mathrm{~cm}$ and $P Q=4.5 \mathrm{~cm}$.
i Find QR.
ii Find the area of triangle PQR.
iii Find height $h$, correct to three significant figures.
b The diagram shows a parallelogram $A B C D$. Angle $B A X$ is a right angle.

$A D=8.7 \mathrm{~cm}$ and $C D=4.2 \mathrm{~cm}$.
The area of triangle $A B X$ is $8.4 \mathrm{~cm}^{2}$.
Find the area of trapezium AXCD.

12 The point $A$ has co-ordinates (7, 2). The point $O$ has co-ordinates $(0,0)$
$B$ is a reflection of A in the line $y=x$.
$C$ is a reflection of $A$ in the $y$ axis.
D is a rotation of A by $90^{\circ}$ clockwise about O .
a Work out the co-ordinates of points $\mathrm{B}, \mathrm{C}$ and D .
b Find the area of quadrilateral $A B C D$.
Use the grid below to help you if you wish, but do all your working in the answer booklet.


13 The diagram shows trapezium $A$ and trapezium $B$.


The height of trapezium $A$ is twice that of trapezium B.
The area of trapezium $A$ is three times that of trapezium $B$.
Find $x$.

14 a Find the area of the equilateral triangle shown below.


The circle below has radius $R \mathrm{~cm}$. Point O is the centre of the circle and the midpoint of one of the sides of a square of side length $R \mathrm{~cm}$.

b Find the total shaded area if $R=27.4$.
c Find $R$ if the total shaded area is $1445 \mathrm{~cm}^{2}$.

